where $(\lambda - c)(\mu - c) = h^2$, $\lambda\mu = l^2$, $\lambda \geqq \mu$. If l is assumed $\geqq c + h$, as is natural, λ and μ are real, unequal, and greater than c. The result of the transformation is

$$A = 2(\lambda - \mu) \int_{\sqrt{\frac{\mu-c}{\lambda-c}}}^{\sqrt{\frac{\mu}{\lambda}}} \frac{\sqrt{(\lambda - c)v^2 - (\mu - c)}}{\sqrt{\mu - \lambda v^2}} \cdot \frac{\lambda v + \mu}{(v + 1)^3} dv.$$

The final result will be simplified if we transform again, putting

$$v = \frac{\sqrt{\mu(\mu - c)}}{\sqrt{\mu(\lambda - c) - (\lambda - \mu)cw^2}},$$

which gives

$$A = 2c(\mu - c) \int_0^1 \frac{(3\lambda c + 3\mu c - 2\lambda\mu - 4c^2)\mu^2 + (\lambda\mu + 5\mu c - 3\mu^2 - 3\lambda c)\mu c w^2 + (\lambda - \mu)\mu c^2 w^4}{(\mu - cw^2)^3 \sqrt{1 - w^2}\sqrt{\mu(\lambda - c) - (\lambda - \mu)cw^2}} \cdot w^2 dw$$

$$+ 2c(\mu - c)\sqrt{\mu(\mu - c)} \int_0^1 \frac{(\lambda + \mu - 4c)\mu - (\lambda - 3\mu)cw^2}{(\mu - cw^2)^3\sqrt{1 - w^2}} \cdot w^2 dw.$$

These integrals are respectively elliptic and circular. On making the necessary reductions, we find, after much calculation,

$$A = c(\mu - c)\sqrt{\frac{\mu}{\lambda - c}} F\left(\sqrt{\frac{(\lambda - \mu)c}{\mu(\lambda - c)}}, \frac{\pi}{2}\right) - c\sqrt{\mu(\lambda - c)} E\left(\sqrt{\frac{(\lambda - \mu)c}{\mu(\lambda - c)}}, \frac{\pi}{2}\right)$$

$$- \frac{c(\mu - c)(\lambda + \mu - c)}{\sqrt{\mu(\lambda - c)}} \Pi\left(-\frac{c}{\mu}, \sqrt{\frac{(\lambda - \mu)c}{\mu(\lambda - c)}}, \frac{\pi}{2}\right) + \frac{\pi}{2} \cdot c(\lambda + \mu - c).$$

2697 [April, 1918]. Proposed by H. S. UHLER, Yale University.

Show how to reduce the left-hand members of the following identities to their respective right members:

$$\sin^2 (x + \tfrac{1}{2}y) - \sin (x + \tfrac{3}{2}y) \sin (x - \tfrac{1}{2}y) = \sin^2 y, \tag{1}$$

$$\sin (x + y) \sin (x + \tfrac{1}{2}y) - \sin x \sin (x + \tfrac{3}{2}y) = \sin \tfrac{1}{2}y \sin y, \tag{2}$$

$$\sin x \sin (x + \tfrac{1}{2}y) - \sin (x - \tfrac{1}{2}y) \sin (x + y) = \sin \tfrac{1}{2}y \sin y. \tag{3}$$

SOLUTION BY POLYCARP HANSEN, St. John's University, Collegeville, Minn.

The terms of the left-hand members can be expressed as follows:

(1)
$$\sin^2 (x + \tfrac{1}{2}y) - \sin (x + \tfrac{3}{2}y) \sin (x - \tfrac{1}{2}y) = \frac{1 - \cos (2x + y)}{2} + \frac{1}{2}[\cos (2x + y) - \cos 2y] = \frac{1 - \cos 2y}{2} = \sin^2 y.$$

(2)
$$\sin (x + y) \sin (x + \tfrac{1}{2}y) - \sin x \sin (x + \tfrac{3}{2}y) = -\tfrac{1}{2}[\cos (2x + \tfrac{3}{2}y) - \cos \tfrac{1}{2}y] + \tfrac{1}{2}[\cos (2x + \tfrac{3}{2}y) - \cos (-\tfrac{3}{2}y)] = \tfrac{1}{2}[\cos \tfrac{1}{2}y - \cos (-\tfrac{3}{2}y)] = \sin \tfrac{1}{2}y \sin y.$$

(3)
$$\sin x \sin (x + \tfrac{1}{2}y) - \sin (x - \tfrac{1}{2}y) \sin (x + y) = -\tfrac{1}{2}[\cos (2x + \tfrac{1}{2}y) - \cos (-\tfrac{1}{2}y)] + \tfrac{1}{2}[\cos (2x + \tfrac{1}{2}y) - \cos (-\tfrac{3}{2}y)] = \sin \tfrac{1}{2}y \sin y.$$

Also solved by R. B. WILDERMUTH, JEROME J. JULIAN, KATHERINE S. ARNOLD, R. M. MATHEWS, H. L. OLSON, H. E. CARLSON, A. T. DINEEN, R. C. COLWELL, and ROGER A. JOHNSON.

2698 [April, 1918]. Proposed by WARREN WEAVER, Throop College of Technology, Pasadena, California.

An urn contains N balls, numbered from 1 to N. Of these n are drawn out and are arranged linearly according to the numbers on each. A certain ball is observed to be the kth in this line. What is the most probable number written on this ball?